

$$n - \text{de driehoeksgetal } D_n = \frac{n(n+1)}{2} = \binom{n+1}{2}$$

$$n - \text{de viervlaksgetal } T_n = \frac{n(n+1)(n+2)}{6} = \binom{n+2}{3}$$

$$\text{Verband: } T_n = \sum_{i=1}^n D_i$$

$$20 = 1 + 3 + 6 + 10$$

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VIERVLAKSGETAL
DRIEHOEKSGETALLEN

Via de formule van STIJEL-PASCAL bekomen we immers dat

$$\begin{aligned}
 20 &= \binom{6}{3} = \binom{5}{3} + \binom{5}{2} \\
 &= \binom{4}{3} + \binom{4}{2} + \binom{5}{2} \\
 &= \binom{3}{3} + \binom{3}{2} + \binom{4}{2} + \binom{5}{2}
 \end{aligned}$$

en $\binom{3}{3} = \binom{2}{2}$, zodat

$$20 = \binom{6}{3} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \binom{5}{2} = 1 + 3 + 6 + 10.$$

TE BEWIJZEN: $\sum_{i=1}^n \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$

Bewijs: $\sum_{i=1}^n \frac{i(i+1)}{2} = \frac{1}{2} \sum_{i=1}^n i^2 + \frac{1}{2} \sum_{i=1}^n i$

$$= \frac{1}{2} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \cdot \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} \left[\frac{2n+1}{6} + \frac{1}{2} \right]$$

$$= \frac{n(n+1)}{2} \cdot \frac{2n+4}{6}$$

$$= \frac{n(n+1)(n+2)}{6}$$