$$\sum_{i=1}^{\infty} \frac{6}{i(i+1)(i+2)} = \frac{3}{2}$$

Bewijs. $$\frac{6}{i(i+1)(i+2)} = \frac{A}{i} + \frac{B}{i+1} + \frac{C}{i+2}$$

$$= \frac{A(i^2+3i+2) + B(i^2+2i) + C(i^2+i)}{i(i+1)(i+2)}$$

dan is $A+B+C = 0$ (coëff. van i^2)

$3A+2B+C=0$ (coëff. van i)

$2A = 6$ (constante termen)

zodat $A=3$, $B=-6$ en $C=3$.

$$\frac{6}{i(i+1)(i+2)} = \frac{3}{i} - \frac{6}{i+1} + \frac{3}{i+2}$$

Dan is

$$\frac{1}{1} = \frac{6}{1.2.3} = \underbrace{\frac{3}{1} - \frac{6}{2} + \frac{3}{3}}_{=0}$$

$$\frac{1}{4} = \frac{6}{2.3.4} = \frac{3}{2} - \frac{6}{3} + \frac{3}{4}$$

$$\frac{1}{10} = \frac{6}{3.4.5} = \frac{3}{3} - \frac{6}{4} + \frac{3}{5}$$

$$\frac{1}{20} = \frac{6}{4.5.6} = \frac{3}{4} - \frac{6}{5} + \frac{3}{6}$$

$$\frac{1}{35} = \frac{6}{5.6.7} = \frac{3}{5} - \frac{6}{6} + \frac{3}{7}$$

$$\vdots$$

$$+\quad \frac{6}{n(n+1)(n+2)} = \frac{3}{n} - \frac{6}{n+1} + \frac{3}{n+2}$$

$$\sum_{i=1}^{n} \frac{6}{i(i+1)(i+2)} = \lim_{n\to\infty}\left(\frac{3}{2} - \frac{6}{n+1} + \frac{3}{n+2}\right)$$

$$= \frac{3}{2}.$$

(bij sommatie vallen de termen in groepjes van drie weg: zie stippellijnen)