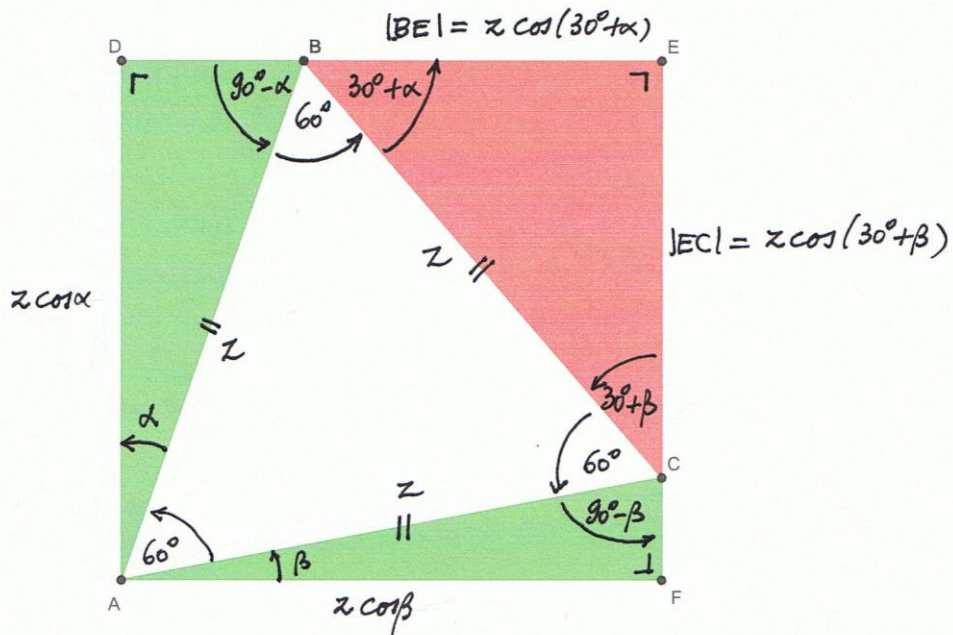


Om een gelijkzijdige driehoek ABC tekent men een rechthoek ADEF zoals op de figuur.
 Op die manier ontstaan de rechthoekige driehoeken CFA, ADB en BEC.
 Dan is opp. $\Delta CFA + \text{opp. } \Delta ADB = \text{opp. } \Delta BEC$.



Bewijs.

$$\text{Opp. } \Delta CFA = \frac{1}{2} \cdot z \cdot z \cos \beta \cdot \sin \beta$$

$$\text{Opp. } \Delta ADB = \frac{1}{2} \cdot z \cdot z \cos \alpha \cdot \sin \alpha$$

$$\begin{aligned} \text{Opp. } \Delta CFA + \text{Opp. } \Delta ADB &= \frac{z^2}{2} (\sin \alpha \cos \alpha + \sin \beta \cos \beta) \\ &= \frac{z^2}{4} (\sin 2\alpha + \sin 2\beta) \\ &= \frac{z^2}{2} \cdot \sin(\alpha + \beta) \cdot \cos(\alpha - \beta) \\ \alpha + \beta &= 30^\circ \\ &= \frac{z^2}{4} \cos(\alpha - \beta). \end{aligned}$$

$$\begin{aligned} \text{Opp. } \Delta BEC &= \frac{1}{2} \cdot z^2 \cos(30^\circ + \alpha) \cos(30^\circ + \beta) \\ &= \frac{z^2}{2} \cdot \frac{1}{2} [\cos(60^\circ + \alpha + \beta) + \cos(30^\circ + \alpha - 30^\circ - \beta)] \\ &= \frac{z^2}{4} [\cos(60^\circ + \alpha + \beta) + \cos(\alpha - \beta)] \\ \alpha + \beta &= 30^\circ \\ &= \frac{z^2}{4} [\cos 90^\circ + \cos(\alpha - \beta)] \\ &= \frac{z^2}{4} \cos(\alpha - \beta) \end{aligned}$$

Q.E.D.