

EIGENSCHAP VAN BREUKEN

$$\frac{a_1}{b_1} < \frac{a_2}{b_2} \text{ (met } b_1 > 0 \text{ en } b_2 > 0) \Rightarrow \frac{a_1}{b_1} < \frac{a_1 + a_2}{b_1 + b_2} < \frac{a_2}{b_2}$$

Bewijs.

Deel 1

$$\begin{aligned} \frac{a_1}{b_1} &< \frac{a_2}{b_2} \\ \Downarrow \\ \frac{a_1 \cdot b_2}{b_1} &< a_2 \\ \Downarrow \\ \frac{a_1 \cdot b_1}{b_1} + \frac{a_1 \cdot b_2}{b_1} &< a_1 + a_2 \\ \Downarrow \\ \frac{a_1 \cdot (b_1 + b_2)}{b_1} &< a_1 + a_2 \\ \Downarrow \\ \frac{a_1}{b_1} &< \frac{a_1 + a_2}{b_1 + b_2}. \end{aligned}$$

Deel 2

$$\begin{aligned} \frac{a_1}{b_1} &< \frac{a_2}{b_2} \\ \Downarrow \\ a_1 &< \frac{b_1 \cdot a_2}{b_2} \\ \Downarrow \\ a_1 + a_2 &< \frac{b_1 \cdot a_2}{b_2} + \frac{b_2 \cdot a_2}{b_2} \\ \Downarrow \\ a_1 + a_2 &< \frac{(b_1 + b_2) \cdot a_2}{b_2} \\ \Downarrow \\ \frac{a_1 + a_2}{b_1 + b_2} &< \frac{a_2}{b_2}. \end{aligned}$$