DE SPOOKRIJ

1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88 …

$$\begin{cases} u_0 = u_1 = u_2 = 1 \\ u_n = u_{n-1} + u_{n-3} \;,\; n \geq 3 \end{cases}$$

$$u_{n+1} = u_n + u_{n-2} \quad (*)$$
$$u_{n+2} = u_{n+1} + u_{n-1}$$

Dan is
$$\frac{u_{n+2}}{u_{n+1}} = \frac{u_{n+1} + u_{n-1}}{u_n + u_{n-2}}$$
$$\overset{(*)}{=} \frac{u_n + u_{n-2} + u_{n-1}}{u_n + u_{n-2}}$$
$$= 1 + \frac{u_{n-1}}{u_n + u_{n-2}}$$
$$= 1 + \frac{1}{\frac{u_n + u_{n-2}}{u_{n-1}}}$$

Dus
$$\frac{u_{n+2}}{u_{n+1}} = 1 + \frac{1}{\frac{u_n}{u_{n-1}} + \frac{1}{\frac{u_{n-1}}{u_{n-2}}}} \quad (**)$$

Wat gebeurt er met de verhouding van twee opeenvolgende termen uit deze rij als n nadert naar $+\infty$?

Stel
$$\lim_{n \to +\infty} \frac{u_{n+2}}{u_{n+1}} = \lim_{n \to +\infty} \frac{u_n}{u_{n-1}} = \lim_{n \to +\infty} \frac{u_{n-1}}{u_{n-2}} = \xi$$

Dan volgt uit (**) dat
$$\xi = 1 + \frac{1}{\xi + \frac{1}{\xi}}$$

zodat
$$\xi = 1 + \frac{\xi}{\xi^2 + 1}$$

of nog: $\xi^3 = \xi^2 + 1$

$$\xi = \sqrt[3]{\frac{29}{54} - \frac{\sqrt{93}}{18}} + \sqrt[3]{\frac{29}{54} + \frac{\sqrt{93}}{18}} + \frac{1}{3} \approx 1,465571231\ldots$$

het 'spookgetal' of supergulden getal

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