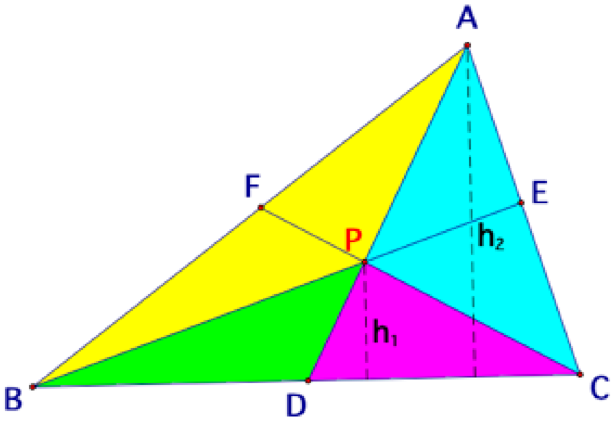
STELLING VAN CEVA.

Als men binnen een driehoek ABC een punt P kiest en als de rechten AP, BP en CP de zijden [BC], [CA] en [AB] respectievelijk snijden in de punten D, E en F, dan is

$$|AF| \cdot |BD| \cdot |CE| = |FB| \cdot |DC| \cdot |EA|$$

Bewijs.



Consider $\triangle BDP$ and $\triangle CDP$. These two triangles have the same height, h_1.

So, $\triangle BDP = \frac{1}{2} * BD * h_1 = \frac{h_1}{2} * BD$ and $\triangle CDP = \frac{1}{2} * DC * h_1 = \frac{h_1}{2} * DC \Rightarrow \frac{h_1}{2} = \frac{\triangle CDP}{DC}$.

Substitute $\frac{h_1}{2}$ of the first equation for $\frac{\triangle CDP}{DC}$ and get $\triangle BDP = \frac{\triangle CDP}{DC} * BD = \frac{BD}{DC} * \triangle CDP$.

Consider $\triangle ABD$ and $\triangle ACD$. These two triangles also have the same height, h_2.

So, $\triangle ABD = \frac{1}{2} * BD * h_2 = \frac{h_2}{2} * BD$ and $\triangle ACD = \frac{1}{2} * DC * h_2 = \frac{h_2}{2} * DC \Rightarrow \frac{h_2}{2} = \frac{\triangle ACD}{DC}$.

Substitute $\frac{h_2}{2}$ of the equation above for $\frac{\triangle ACD}{DC}$ and get $\triangle ABD = \frac{\triangle ACD}{DC} * BD = \frac{BD}{DC} * \triangle ACD$.

Consider $\triangle ABP$ and $\triangle ACP$. $\triangle ABP = \triangle ABD - \triangle BDP$ and $\triangle ACP = \triangle ACD - \triangle CDP$.

The ratio of $\frac{\triangle ABP}{\triangle ACP} = \frac{\triangle ABD - \triangle BDP}{\triangle ACD - \triangle CDP} = \frac{\frac{BD}{DC} * \triangle ACD - \frac{BD}{DC} * \triangle CDP}{\triangle ACD - \triangle CDP} = \frac{BD}{DC}$.

Similar relationships, $\frac{\triangle ACP}{\triangle BCP} = \frac{AF}{FB}$ and $\frac{\triangle BCP}{\triangle ABP} = \frac{EC}{EA}$, can be derived by repeating the procedures above for $\triangle ACP$, $\triangle CBP$, $\triangle BCP$, and $\triangle BAP$.

Therefore, $\frac{AF}{FB} * \frac{BD}{DC} * \frac{EC}{EA} = \frac{(AF)(BD)(EC)}{(FB)(DC)(EA)} = \frac{\triangle ACP}{\triangle BCP} * \frac{\triangle ABP}{\triangle ACP} * \frac{\triangle BCP}{\triangle ABP} = 1$.

This complete the proof that (AF)(BD)(EC) = (FB)(DC)(EA).

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Ceva's Theorem

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