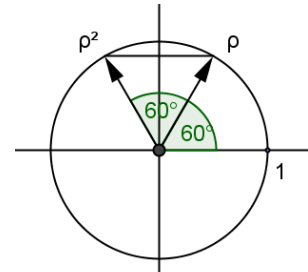


## Antwoorden

- 1 a  $\alpha' = -\bar{\alpha}$  ( $\bar{\alpha}$  is de notatie voor geconjugeerde)  
 b  $\alpha' = -\alpha$   
 c  $\alpha' = \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}\right) \cdot \alpha$   
 d  $\alpha' = \left(\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) \cdot \alpha$   
 e  $\alpha' = \left(-\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right) \cdot \alpha$



Draaien over  $120^\circ$  is twee keer draaien over  $60^\circ$ .

Complex: vermenigvuldigen met  $\rho^2$ .

Zowel rekenkundig als in de tekening zie je  $\rho^2 = \rho - 1$

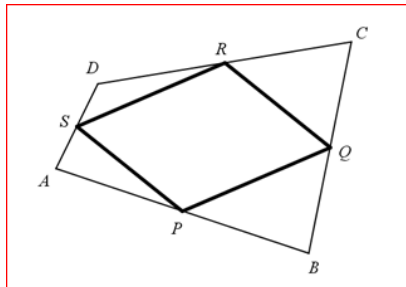
2  $\gamma = \frac{2}{3}\alpha + \frac{1}{3}\beta$  en  $\delta = \frac{1}{3}\alpha + \frac{2}{3}\beta$

3 a  $\delta - \varepsilon = \frac{1}{2}(\beta + \gamma) - \frac{1}{2}(\alpha + \gamma) = \frac{1}{2}(\beta - \alpha)$   
 dus  $DE \parallel AB$  en  $DE = \frac{1}{2}AB$ .

a)  $AZ = \frac{2}{3}AD$ , uit opgave 2 volgt dan:

$$\zeta = \frac{1}{3}\alpha + \frac{2}{3}\delta = \frac{1}{3}\alpha + \frac{2}{3} \cdot \frac{1}{2}(\beta + \gamma) = \frac{1}{3}(\alpha + \beta + \gamma)$$

4



$$\rho - \sigma = \frac{1}{2}(\gamma + \delta) - \frac{1}{2}(\alpha + \delta) = \frac{1}{2}(\gamma - \alpha)$$

$$\xi - \pi = \frac{1}{2}(\beta + \gamma) - \frac{1}{2}(\alpha + \beta) = \frac{1}{2}(\gamma - \alpha); \text{ (bij } Q \text{ neem je de letter } \xi \text{)}$$

$SR \parallel PQ$  en  $SR = PQ \Rightarrow PQRS$  is een parallellogram.

5  $\overline{BD} = \overline{BA} + \overline{BC}$ , dus  $\delta - \beta = \alpha - \beta + \gamma - \beta$  en dus  $\delta = \alpha + \gamma - \beta$ .

6  $\beta' - \alpha = i(\beta - \alpha)$ , dus  $\beta' = \alpha + i(\beta - \alpha)$

7 Te bewijzen:  $i(\varepsilon - \mu) = \delta - \mu$

$$\mu = \frac{1}{2}(\alpha + \beta)$$

Uit opgave 6 volgt:

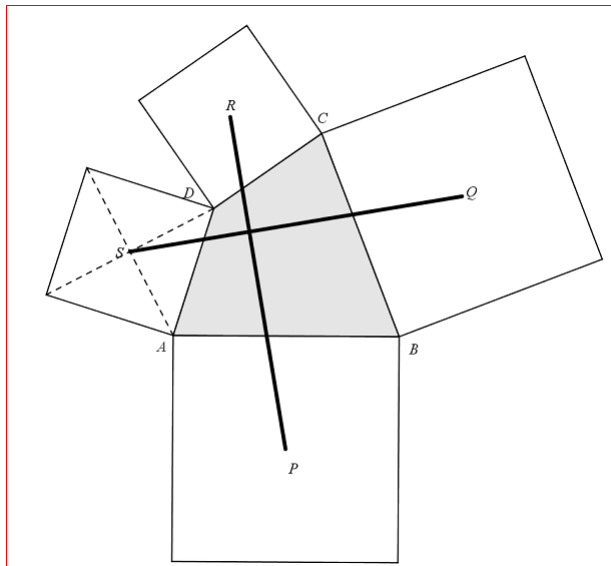
$$\varepsilon = \frac{1}{2}(\beta + \gamma) + i(\beta - \frac{1}{2}(\beta + \gamma)) = \frac{1}{2}(\beta + \gamma) + \frac{1}{2}i(\beta - \gamma)$$

$$\delta = \frac{1}{2}(\alpha + \gamma) + \frac{1}{2}i(\gamma - \alpha)$$

$$\varepsilon - \mu = \frac{1}{2}(\gamma - \alpha) + \frac{1}{2}i(\beta - \gamma) \text{ en } \delta - \mu = \frac{1}{2}(\gamma - \beta) + \frac{1}{2}i(\gamma - \alpha)$$

$$i(\varepsilon - \mu) = \frac{1}{2}i(\gamma - \alpha) + \frac{1}{2}i^2(\beta - \gamma) = \frac{1}{2}(\gamma - \beta) + \frac{1}{2}i(\gamma - \alpha) = \delta - \mu$$

8



Te bewijzen:  $i(\xi - \sigma) = \rho - \pi$

$$\sigma = \frac{1}{2}(\alpha + \delta) + \frac{1}{2}i(\delta - \alpha)$$

$$\rho = \frac{1}{2}(\gamma + \delta) + \frac{1}{2}i(\gamma - \delta)$$

$$\xi = \frac{1}{2}(\beta + \gamma) + \frac{1}{2}i(\beta - \gamma)$$

$$\pi = \frac{1}{2}(\alpha + \beta) + \frac{1}{2}i(\alpha - \beta)$$

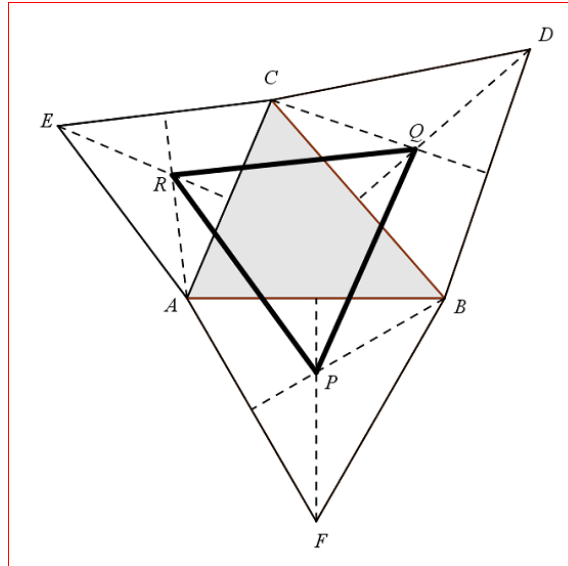
$$\xi - \sigma = \frac{1}{2}(\beta + \gamma - \alpha - \delta) + \frac{1}{2}i(\alpha + \beta - \gamma - \delta)$$

$$\rho - \pi = \frac{1}{2}(\gamma + \delta - \alpha - \beta) + \frac{1}{2}i(\beta + \gamma - \alpha - \delta)$$

$$i(\xi - \sigma) = \frac{1}{2}i(\beta + \gamma - \alpha - \delta) + \frac{1}{2}i^2(\alpha + \beta - \gamma - \delta)$$

$$= \frac{1}{2}(\gamma + \delta - \alpha - \beta) + \frac{1}{2}i(\beta + \gamma - \alpha - \delta) = \rho - \pi$$

9



Te bewijzen:  $\rho(\xi - \pi) = \tau - \pi$

(vanwege de verwarring met de rotatie  $\rho$  (zie opgave 1) kiezen we bij het punt  $R$  de tau ( $\tau$ ))

$\pi = \frac{1}{3}(\alpha + \beta + \varphi)$ ,  $\xi = \frac{1}{3}(\beta + \gamma + \delta)$  en  $\tau = \frac{1}{3}(\alpha + \lambda + \varepsilon)$  met  
 $\varphi = \beta + \rho(\alpha - \beta)$ ,  $\delta = \gamma + \rho(\beta - \gamma)$  en  $\varepsilon = \alpha + \rho(\gamma - \alpha)$

Om geen last te hebben van de herhaald voorkomende  $\frac{1}{3}$  bewijzen we

$$3\rho(\xi - \pi) = 3(\tau - \pi)$$

$$3(\tau - \pi) = \gamma + \varepsilon - \beta - \varphi = (\alpha + \gamma - 2\beta) + \rho(\beta + \gamma - 2\alpha)$$

$$3(\xi - \pi) = \gamma + \delta - \alpha - \varphi = (2\gamma - \alpha - \beta) + \rho(2\beta - \alpha - \gamma)$$

$$\begin{aligned} 3\rho(\xi - \pi) &= \rho(2\gamma - \alpha - \beta) + \rho^2(2\beta - \alpha - \gamma) \quad (\text{Bedenk: } \rho^2 = \rho - 1) \\ &= \rho(2\gamma - \alpha - \beta) + \rho(2\beta - \alpha - \gamma) - (2\beta - \alpha - \gamma) \\ &= (\alpha + \gamma - 2\beta) + \rho(\beta + \gamma - 2\alpha) = 3(\tau - \pi) \end{aligned}$$