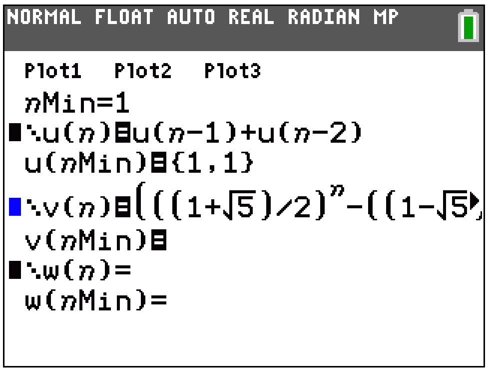
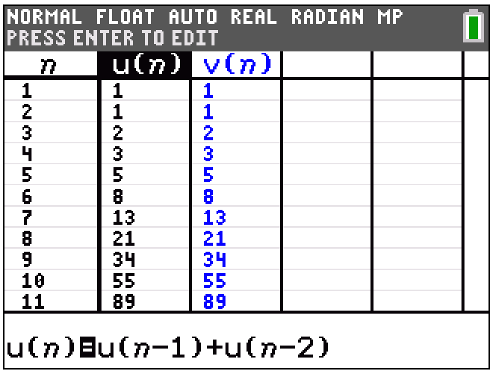
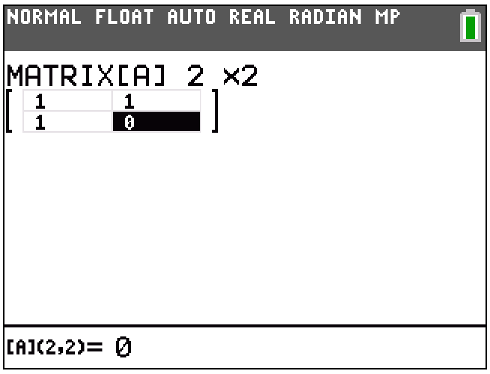
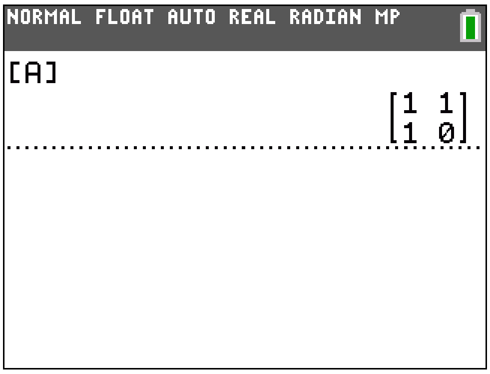
FIBONACCI EN PELL

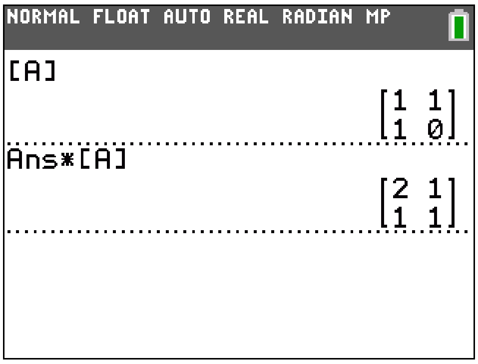
RIJ VAN FIBONACCI

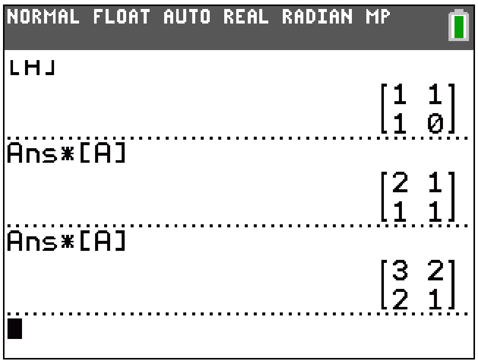












Vaststelling.

$$A^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix},$$ waarbij F_n het n-de Fibonaccigetal is.

Bewijs door volledige inductie.

- Basisstap. De formule klopt voor n = 1 (met de afspraak dat $F_0 = 0$).
- Inductiestap.
  Stel dat

$$A^k = \begin{bmatrix} F_{k+1} & F_k \\ F_k & F_{k-1} \end{bmatrix},$$

Dan is

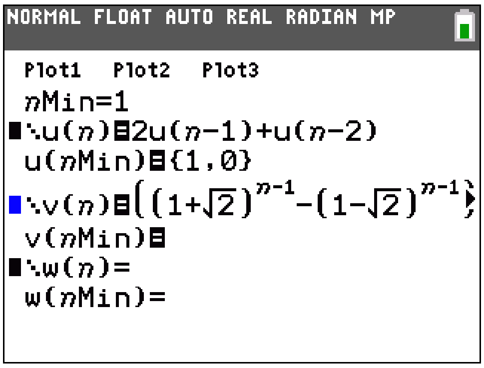
$$A^{k+1} = \begin{bmatrix} F_{k+1} & F_k \\ F_k & F_{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} F_{k+1} + F_k & F_{k+1} \\ F_k + F_{k-1} & F_k \end{bmatrix} = \begin{bmatrix} F_{k+2} & F_{k+1} \\ F_{k+1} & F_k \end{bmatrix}.$$

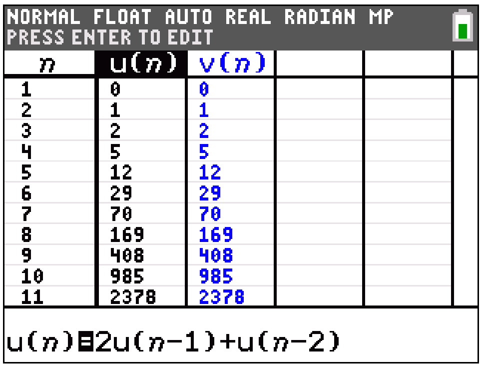
<u>RIJ VAN PELL</u>

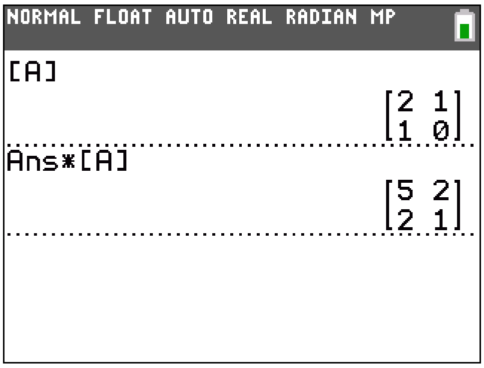
0, 1, 2, 5, 12, 29, 70, 169 …

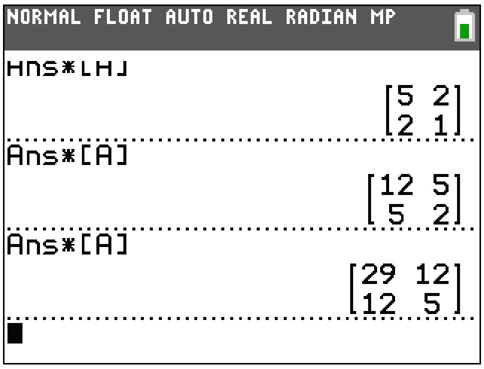
Voor het n-de Pellgetal bestaat er ook een expliciet voorschrift:

$$P_n = \frac{(1+\sqrt{2})^{n-1} - (1-\sqrt{2})^{n-1}}{2\sqrt{2}}\ .$$









$A^n = \begin{bmatrix} P_{n+1} & P_n \\ P_n & P_{n-1} \end{bmatrix}$, waarbij P_n het n-de Pellgetal is.

Dit kan eveneens via volledige inductie bewezen worden.

Meer informatie op Wikipedia (Pellgetal) en www.gnomon.bloggen.be (zoekopdracht: Pell).