Oplossingen

1 (a) ja

(b) ja

(c) ja

2 (a) $y' = e^x$

(b) $y' = y$

(c) $y'' = 0$

3 (a) $y = \dfrac{x^3}{Cx^3 + 1}$ en $y = 0$

(b) $y = \left(\dfrac{x^2}{4} + c\right)^2$ en $y = 0$

(c) $y = Cx^3$ en $y = C|x|^3$

(d) $y = \dfrac{1}{\arctan(e^x) + C}$ en $y = 0$

(e) $y = C \sin x$

(f) $y = a + \dfrac{Cx}{1 + ax}$

(g) $y = \pm\sqrt{C - x^2}$

4 (a) $\cot^2 y = \tan^2 x + C$

(b) $x = \dfrac{Cy}{\sqrt{1 + y^2}}$; $y = 0$

(c) $x^2 + y^2 = \ln(Cx^2)$

(d) $\tan y = C(1 - e^x)^3$

(e) $y = \dfrac{-4}{x^2 + C}$

(f) $y = Ce^{5x}$

(g) $y = Ce^{-\frac{1}{x}}$ met $C \neq 0$

5 (a) $2e^{\frac{1}{2}y^2} = \sqrt{e}(1 + e^x)$

(b) $y = \sqrt{2e^x - 1}$

(c) $1 + y^2 = \dfrac{2}{1 - x^2}$

(d) $y = -\sqrt{2 + 2\cos x}$

(e) $y = 1$

(f) $y = e^{-\frac{1}{3}(x^3 + 3x + 4)}$

6 $x^2 - y^2 = C$

De vergelijking die moet worden opgelost is

$$yy' = x.$$

7 (a) $y = Cx$ en $x = 0$

De vergelijking die moet worden opgelost is

$$y' = \frac{y}{x}.$$

(b) $x^2 + 2y^2 = 2C$

(c) $y^2 = -2x + C$

8 -60 °C

9 (a) $P(t) = \dfrac{MP_0e^{Mkt}}{M - P_0 + P_0e^{Mkt}}$

(b) M

(c) wereldbevolking op lange termijn

(d) 8,23 miljard, 9,42 miljard

10 7 062

11 (c) De lijkwade is 583 jaar oud.

12 4,2%

De hoeveelheid radioactief isotoop wordt gegeven door

$$Q = Q_0(\frac{1}{2})^{\frac{t}{1600}}.$$

13 35,2 s.

De vergelijking die moet worden opgelost, is

$$\pi(h^2 - 2h)dh = \pi\frac{1}{100}vdt.$$

15 $y = Cx$ of $y = \dfrac{C}{x}$

De vergelijking die moet worden opgelost is

$$\sqrt{x^2 + y^2} = \sqrt{y^2 + \left(\frac{y}{y'}\right)^2}$$

16 (a) $y = x \ln \dfrac{C}{x}$

(b) $y = \dfrac{C}{x} - \dfrac{x}{2}$

(c) $x = Ce^{\frac{x}{y}}$

(d) $\sqrt{\dfrac{x}{y}} + \ln|y| = C$

(e) $y = \dfrac{Cx^2}{2} - \dfrac{1}{2C}$

(f) $(x^2+y^2)^3(x+y)^2 = C$

17 (a) $(x-C)^2 - y^2 = C^2$

(b) $(x-2)^2 - y^2 = 4; y = \pm x$

18 $x^2 + y^2 = Cy$

19 logaritmische spiraal $r = e^{c-\theta}$

20 (a) $(x+y-1)^3 = C(x-y+3)$

(b) $3x + y + 2\ln|x+y-1| = C$

(c) $\ln|4x+8y+5| + 8y - 4x = C$

21 $y = Cx - x\ln|x|$

22 (a) $\dfrac{x^3}{3} + xy^2 + x^2 = C$

(b) $\dfrac{x^4}{4} - \dfrac{3x^2y^2}{2} + 2x + \dfrac{y^3}{3} = C$

(c) $x^2 + y^2 - 2\arctan\dfrac{y}{x} = C$

23 $x^2 - y^2 = Cy^3$

24 1 of 2 leerlingen

25 (a) $\ln|x| - \dfrac{y^2}{x} = C$

(b) $\dfrac{1}{y}\ln x + \dfrac{1}{2}y^2 = C$

(c) $(x\sin y + y\cos y - \sin y)e^x = C$

26 (a) $y = Cx + x^2$

(b) $y = \dfrac{1}{6}x^4 + \dfrac{C}{x^2}$

(c) $x = Cy^2 - \dfrac{1}{y}$

27 (a) $y = \dfrac{e^x}{x} + \dfrac{ab - e^a}{x}$

(b) $y = \dfrac{1}{2}(x\sqrt{1-x^2} + \arcsin x)\sqrt{\dfrac{1+x}{1-x}}$

(c) $y = \dfrac{x}{\cos x}$

28 $xy = Cy^2 + a^2$

29 $4\,8$ kg zand (dit gebeurt na 20 minuten)

30 $I(t) = \dfrac{609}{101}e^{-20t} + \dfrac{30}{101}\sin t - \dfrac{3}{101}\cos 2t$

31 (a) $y(x^2 + Cx) = 1$

(b) $y^2 = x\ln\dfrac{C}{x}$

(c) $x^2(y + Cy^2) = 1$

32 $x^2 + y^2 - Cy + 1 = 0$

33 (a) $y = 1 + \dfrac{2}{2x-1}$

(b) $y = \dfrac{1}{x} + \dfrac{2}{3x^3 - x}$

34 (a) $y = e^{-x} + 3xe^{-x}$

(b) $y = 2\sqrt{3}e^{-\frac{x}{2}}\sin\dfrac{\sqrt{3}x}{2}$

(c) $y = \dfrac{1}{2}x^3 + 2x^2 - \dfrac{1}{2}x + 2$

(d) $y = \dfrac{8}{3} + \dfrac{e^{-6}}{3}e^{3x}$

(e) $y = C_1\cos 2x + C_2\sin 2x + C_3 x\cos 2x + C_4\sin 2x$

(f) $y = C_1e^x + C_2\cos x + C_3\sin x$

35 (a) $y = C_1 x^{3+2\sqrt{2}} + C_2 x^{3-2\sqrt{2}}$

(b) $y = C_1 x^{-\frac{1}{4}} \cos\left(\frac{\sqrt{7}}{4}\ln x\right) + C_2 x^{-\frac{1}{4}} \sin\left(\frac{\sqrt{7}}{4}\ln x\right)$

(c) $y = C_1 + C_2 x^{\frac{5}{2}} \cos\left(\frac{\sqrt{3}}{2}\ln x\right) + C_2 x^{\frac{5}{2}} \sin\left(\frac{\sqrt{3}}{2}\ln x\right)$

36 (a) $y = C_1 x^2 e^x + C_2 e^x$

(b) $y = \frac{x}{2}(\ln x)^2 + C_1 x \ln x + C_2 x$

37 (a) $y = \frac{1}{3}e^x - \frac{1}{3}e^{-x} + \frac{1}{3}e^{2x}$

(b) $y = 2\cos 2x + \frac{1}{2}\sin 2x - \frac{1}{24}\sin 4x$

(c) $y = C_1 e^{2x} + C_2 x e^{2x} + \frac{1}{8}e^{4x}$

(d) $C_1 + C_2 \cos x + C_3 \sin x + \ln|\sec x + \tan x| - x\cos x + \sin x \ln|\cos x|$

(e) $y = C_1 x^2 + C_2 x^2 (\ln x)^2 + \frac{1}{2}x^2(\ln x)^2$

(f) $y = C_1 \frac{1}{x^2} + C_2 \frac{\ln x}{x^2} + \frac{\ln x - 1}{4}$

(g) $y = C - 1e^x + C - 2e^{-x} + \frac{1}{3}e^{2x} - \frac{1}{2}\cos x$

(h) $y = \left(-\frac{1}{36}xe^{6x} + \frac{1}{216}e^6 x\right)e^{-3x} + \left(\frac{1}{6}x^2 - \frac{1}{36}x\right)e^{3x}$

39 (a) $x^2 + y^4 = Cy^2$

(b) $y = \frac{x}{x^2 + C}$

(c) $xy\left(C - \frac{1}{2}\ln^2 x\right)$

(d) $y = Cx + C\ln C$ en $y = e^{-x-1}$

(e) $3y + \ln\frac{|x^3 - 1|}{(y+1)^6} = C$

(f) $y = x^2(1 + Ce^{\frac{1}{x}})$

(g) $x = y^2(C - e^{-y})$

(h) $y = Ce^{-\sin x} + \sin x - 1$

(i) $y = ax + C\sqrt{1 - x^2}$

(j) $y = \frac{x}{x+1}(x + \ln|x| + C)$

(k) $x = Ce^{\sin y} - 2a(1 + \sin y)$

(l) $2\arctan\frac{y-1}{2x} = \ln|x| + C$

(m) $x^2 = 1 - \frac{2}{y} + Ce^{-\frac{2}{y}}$

(n) $x^3 = Ce^y - y - 2$

(o) $y = x\arcsin(Cx)$

(p) $y^2 = Ce^{-2x} + \frac{2}{5}\sin x + \frac{4}{5}\cos x$

(q) $xy = C(y-1)$

(r) $\sqrt{x^2 + y^2} - \frac{x}{y} = C$

(s) $xe^y - y^2 = C$

(t) $y = xe^{Cx}$

(u) $2e^x - y^4 = Cy^2$

(v) $y^2 + Ce^{-\frac{y^2}{2}} + \frac{1}{x} - 2 = 0$

(w) $x^2 y = Ce^{\frac{y}{a}}$

(x) $x + \frac{x}{y} = C$

(y) $y = C\sin x - a$